

LATTICES  
OF EQUI-TRANSLATION  
SUBGROUPS OF THE SPACE GROUPS

by

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A contribution to a  
work by

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A small first edition appeared in January 1965. In summer 1966 I learned that J. Neubüser (Kiel) and H. Wondratschek (Karlsruhe) had determined both the equi-translation (zellengleich) - as is done here - and the equi-class (klassengleich) subgroups of the space groups. Their results are recorded in a report called "Tables of the maximal subgroups of the space groups". Prof. Wondratschek kindly undertook to compare his results with mine. He not only corrected numerous drawing and printing errors but also some factual mistakes, for which I am very grateful to him. However, this is a new edition and I fear that some new errors may have crept in. I would be thankful to anyone pointing out any error in these tables.

Geneva, January 1968

No of space group	Schoenflies symbol	Standard short symbol	Page
1	$C_1^1$	P1	--
2	$C_i^1$	$\bar{P}1$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51
3	$C_2^1$	P2	1, 2, 3, 4, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 47, 48
4	$C_2^2$	$P2_1$	11, 16, 17, 18, 19, 24, 25, 26, 27, 32, 35, 36, 37, 40, 41, 42, 43, 44, 45, 46
5	$C_2^3$	B2	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 46, 47, 48, 49, 50, 51
6	$C_s^1$	Pm	1, 3, 12, 13, 16, 17, 18, 20, 21, 24, 25, 26, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47
7	$C_s^2$	Pb	2, 4, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48
8	$C_s^3$	Bm	1, 4, 5, 6, 7, 9, 12, 14, 16, 18, 21, 23, 25, 27, 28, 29, 30, 33, 35, 36, 46, 47, 48, 49, 50, 51
9	$C_s^4$	Bb	2, 3, 6, 7, 8, 10, 13, 15, 17, 19, 20, 22, 24, 26, 29, 30, 31, 34, 35, 36, 46, 47, 48, 49, 50, 51

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10	$C_{2h}^1$	P2/m	1, 3, 12, 13, 16, 17, 20, 21, 24, 25, 33, 34, 38, 39, 40, 41, 42, 43, 47
11	$C_{2h}^2$	$P2_1/m$	18, 26, 35, 36, 40, 43, 44, 45, 46
12	$C_{2h}^3$	B2/m	1, 4, 5, 6, 7, 9, 12, 14, 16, 18, 21, 23, 25, 27, 28, 29, 30, 33, 35, 36, 46, 47, 48, 49, 50, 51
13	$C_{2h}^4$	P2/b	2, 4, 13, 14, 15, 18, 19, 21, 22, 23, 26, 27, 38, 39, 40, 41, 42, 43, 44, 48
14	$C_{2h}^5$	$P2_1/b$	11, 16, 17, 19, 24, 25, 27, 40, 41, 42, 43, 44, 45, 46
15	$C_{2h}^6$	B2/b	2, 3, 6, 7, 8, 10, 13, 15, 17, 19, 20, 22, 24, 26, 29, 30, 31, 34, 35, 36, 46, 47, 48, 49, 50, 51
16	$D_2^1$	P222	1, 2, 3, 4, 12, 13, 14, 15, 20, 21, 22, 23, 38, 39
17	$D_2^2$	$P222_1$	32, 40, 41
18	$D_2^3$	$P2_1^2_1^2$	16, 17, 18, 19, 24, 25, 26, 27, 42, 43, 44
19	$D_2^4$	$P2_1^2_1^2_1$	11, 45
20	$D_2^5$	$C222_1$	11, 32, 35, 36, 37, 46

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21	$D_2^6$	C222	1, 2, 3, 4, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 33, 34, 37, 47, 48
22	$D_2^7$	F222	5, 6, 7, 8, 9, 10, 28, 29, 30, 31, 49
23	$D_2^8$	I222	5, 6, 9, 28, 29, 50
24	$D_2^9$	$I2_1^2 2_1^2 2_1^2$	7, 8, 10, 30, 31, 51
25	$C_{2v}^1$	Pmm2	1, 3, 12, 18, 20, 26, 38, 40, 44
26	$C_{2v}^2$	Pmc2 <sub>1</sub>	16, 24, 40, 42, 43, 45
27	$C_{2v}^3$	Pcc2	13, 19, 21, 27, 39, 41, 42
28	$C_{2v}^4$	Pma2	13, 21, 39, 40, 41, 43
29	$C_{2v}^5$	Pca2 <sub>1</sub>	11, 41, 43, 44, 45
30	$C_{2v}^6$	Pnc2	14, 22, 39, 40, 41, 44
31	$C_{2v}^7$	Pmn2 <sub>1</sub>	17, 18, 25, 26, 41, 43, 44, 45
32	$C_{2v}^8$	Pba2	14, 16, 22, 24, 39, 41, 42
33	$C_{2v}^9$	Pna2 <sub>1</sub>	19, 27, 40, 42, 44, 45
34	$C_{2v}^{10}$	Pnn2	2, 4, 15, 17, 23, 25, 38, 40, 43

No of space group	Schoenflies symbol	Standard short symbol	Page
35	$C_{2v}^{11}$	Cmm2	1, 4, 12, 14, 16, 18, 21, 23, 25, 27, 33, 47, 48
36	$C_{2v}^{12}$	Cmc2 <sub>1</sub>	35, 36, 46
37	$C_{2v}^{13}$	Ccc2	2, 3, 13, 15, 17, 19, 20, 22, 24, 26, 34, 47, 48
38	$C_{2v}^{14}$	Amm2	1, 12, 16, 21, 25, 33, 35, 36, 46, 47
39	$C_{2v}^{15}$	Abm2	4, 14, 18, 23, 27, 46, 48
40	$C_{2v}^{16}$	Ama2	3, 13, 17, 20, 24, 34, 35, 36, 46, 47
41	$C_{2v}^{17}$	Aba2	2, 15, 19, 22, 26, 46, 48
42	$C_{2v}^{18}$	Fmm2	5, 6, 9, 28, 29, 49
43	$C_{2v}^{19}$	Fdd2	7, 8, 10, 30, 31, 49
44	$C_{2v}^{20}$	Imm2	5, 7, 9, 28, 30, 50, 51
45	$C_{2v}^{21}$	Iba2	6, 8, 10, 29, 31, 50, 51
46	$C_{2v}^{22}$	Ima2	6, 7, 29, 30, 50, 51
47	$D_{2h}^1$	Pmmm	1, 3, 12, 20, 38
48	$D_{2h}^2$	Pnnn	2, 4, 15, 23, 38
49	$D_{2h}^3$	Pccm	13, 21, 39

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50	$D_{2h}^4$	Pban	14, 22, 39
51	$D_{2h}^5$	Pmma	40
52	$D_{2h}^6$	Pnna	40
53	$D_{2h}^7$	Pmna	41
54	$D_{2h}^8$	Pcca	41
55	$D_{2h}^9$	Pbam	16, 24, 42
56	$D_{2h}^{10}$	Pccn	19, 27, 42
57	$D_{2h}^{11}$	Pbcm	43
58	$D_{2h}^{12}$	Pnnm	17, 25, 43
59	$D_{2h}^{13}$	Pmmn	18, 26, 44
60	$D_{2h}^{14}$	Pbcn	44
61	$D_{2h}^{15}$	Pbca	11, 45
62	$D_{2h}^{16}$	Pnma	45
63	$D_{2h}^{17}$	Cmcm	35, 36, 46
64	$D_{2h}^{18}$	Cmca	46
65	$D_{2h}^{19}$	Cmmm	1, 12, 16, 21, 25, 33, 47

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66	$D_{2h}^{20}$	Cccm	3, 13, 17, 20, 24, 34, 47
67	$D_{2h}^{21}$	Cmma	4, 14, 18, 23, 27, 48
68	$D_{2h}^{22}$	Ccca	2, 15, 19, 22, 26, 48
69	$D_{2h}^{23}$	Fmmm	5, 6, 9, 28, 29, 49
70	$D_{2h}^{24}$	Fddd	7, 8, 10, 30, 31, 49
71	$D_{2h}^{25}$	Immm	5, 9, 28, 50
72	$D_{2h}^{26}$	Ibam	6, 29, 50
73	$D_{2h}^{27}$	Ibca	8, 10, 31, 51
74	$D_{2h}^{28}$	Imma	7, 30, 51
75	$C_4^1$	P4	1, 2, 12, 13, 14, 15, 16, 17, 18, 19
76	$C_4^2$	$P4_1$	11, 32
77	$C_4^3$	$P4_2$	3, 4, 20, 21, 22, 23, 24, 25, 26, 27
78	$C_4^4$	$P4_3$	11, 32
79	$C_4^5$	I4	5, 6, 9, 28, 29
80	$C_4^6$	$I4_1$	7, 8, 10, 30, 31



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81	$S_4^1$	$\bar{P}4$	1, 2, 3, 4, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27
82	$S_4^2$	$\bar{I}4$	5, 6, 7, 8, 9, 10, 28, 29, 30, 31
83	$C_{4h}^1$	$P4/m$	1, 12, 13, 16, 17
84	$C_{4h}^2$	$P4_2/m$	3, 20, 21, 24, 25
85	$C_{4h}^3$	$P4/n$	2, 14, 15, 18, 19
86	$C_{4h}^4$	$P4_2/n$	4, 22, 23, 26, 27
87	$C_{4h}^5$	$I4/m$	5, 6, 9, 28, 29
88	$C_{4h}^6$	$I4_1/a$	7, 8, 10, 30, 31
89	$D_4^1$	$P422$	1, 2, 12, 13, 14, 15
90	$D_4^2$	$P42_1^2$	16, 17, 18, 19
91	$D_4^3$	$P4_1^2$	32
92	$D_4^4$	$P4_1^2$	11
93	$D_4^5$	$P4_2^2$	3, 4, 20, 21, 22, 23
94	$D_4^6$	$P4_2^2$	24, 25, 26, 27
95	$D_4^7$	$P4_3^2$	32

No of space group	Schoenflies symbol	Standard short symbol	Page
81	$S_4^1$	$\bar{P}4$	1, 2, 3, 4, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27
82	$S_4^2$	$\bar{I}4$	5, 6, 7, 8, 9, 10, 28, 29, 30, 31
83	$C_{4h}^1$	$P4/m$	1, 12, 13, 16, 17
84	$C_{4h}^2$	$P4_2/m$	3, 20, 21, 24, 25
85	$C_{4h}^3$	$P4/n$	2, 14, 15, 18, 19
86	$C_{4h}^4$	$P4_2/n$	4, 22, 23, 26, 27
87	$C_{4h}^5$	$I4/m$	5, 6, 9, 28, 29
88	$C_{4h}^6$	$I4_1/a$	7, 8, 10, 30, 31
89	$D_4^1$	$P422$	1, 2, 12, 13, 14, 15
90	$D_4^2$	$P42_12$	16, 17, 18, 19
91	$D_4^3$	$P4_122$	32
92	$D_4^4$	$P4_12_12$	11
93	$D_4^5$	$P4_222$	3, 4, 20, 21, 22, 23
94	$D_4^6$	$P4_22_12$	24, 25, 26, 27
95	$D_4^7$	$P4_322$	32

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96	$D_4^8$	$P4_3^2 1^2$	11
97	$D_4^9$	I422	5, 6, 9, 28, 29
98	$D_4^{10}$	$I4_1 22$	7, 8, 10, 30, 31
99	$C_{4v}^1$	P4mm	1, 12, 18
100	$C_{4v}^2$	P4bm	14, 16
101	$C_{4v}^3$	$P4_2 cm$	21, 27
102	$C_{4v}^4$	$P4_2 nm$	4, 23, 25
103	$C_{4v}^5$	P4cc	13, 19
104	$C_{4v}^6$	P4nc	2, 15, 17
105	$C_{4v}^7$	$P4_2 mc$	3, 20, 26
106	$C_{4v}^8$	$P4_2 bc$	22, 24
107	$C_{4v}^9$	I4mm	5, 9, 28
108	$C_{4v}^{10}$	I4cm	6, 28
109	$C_{4v}^{11}$	$I4_1 md$	7, 30
110	$C_{4v}^{12}$	$I4_1 cd$	8, 10, 31
111	$D_{2d}^1$	$\bar{P}42m$	1, 4, 12, 14, 21, 23

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112	$D_{2d}^2$	$\bar{P}4_2c$	2, 3, 13, 15, 20, 22
113	$D_{2d}^3$	$\bar{P}4_2_1m$	16, 18, 25, 27
114	$D_{2d}^4$	$\bar{P}4_2_1c$	17, 19, 24, 26
115	$D_{2d}^5$	$\bar{P}4m2$	1, 3, 12, 18, 20, 26
116	$D_{2d}^6$	$\bar{P}4c2$	13, 19, 21, 27
117	$D_{2d}^7$	$\bar{P}4b2$	14, 16, 22, 24
118	$D_{2d}^8$	$\bar{P}4n2$	2, 4, 15, 17, 23, 25
119	$D_{2d}^9$	$\bar{I}4m2$	5, 7, 9, 28, 30
120	$D_{2d}^{10}$	$\bar{I}4c2$	6, 8, 10, 29, 31
121	$D_{2d}^{11}$	$\bar{I}4_2m$	5, 6, 9, 28, 29
122	$D_{2d}^{12}$	$\bar{I}4_2d$	7, 8, 10, 30, 31
123	$D_{4h}^1$	$P4/mmm$	1, 12
124	$D_{4h}^2$	$P4/mcc$	13
125	$D_{4h}^3$	$P4/nbm$	14
126	$D_{4h}^4$	$P4/nnc$	2, 15
127	$D_{4h}^5$	$P4/mbm$	16

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128	$D_{4h}^6$	$P4/mnc$	17
129	$D_{4h}^7$	$P4/nmm$	18
130	$D_{4h}^8$	$P4/ncc$	19
131	$D_{4h}^9$	$P4_2/mmc$	3, 20
132	$D_{4h}^{10}$	$P4_2/mcm$	21
133	$D_{4h}^{11}$	$P4_2/nbc$	22
134	$D_{4h}^{12}$	$P4_2/nm$	4, 23
135	$D_{4h}^{13}$	$P4_2/mbc$	24
136	$D_{4h}^{14}$	$P4_2/mnm$	25
137	$D_{4h}^{15}$	$P4_2/nmc$	26
138	$D_{4h}^{16}$	$P4_2/ncm$	27
139	$D_{4h}^{17}$	$I4/mmm$	5, 9, 28
140	$D_{4h}^{18}$	$I4/mcm$	6, 29
141	$D_{4h}^{19}$	$I4_1/amd$	7, 30
142	$D_{4h}^{20}$	$I4_1/acd$	8, 10, 31
143	$C_3^1$	$P3$	33, 34, 35, 36

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144	$C_3^2$	$P3_1$	37
145	$C_3^3$	$P3_2$	37
146	$C_3^4$	R3	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
147	$C_{3i}^1$	$\bar{P}3$	33, 34, 35, 36
148	$C_{3i}^2$	$\bar{R}3$	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
149	$D_3^1$	P312	33, 34, 35, 36
150	$D_3^2$	P321	33, 34, 35, 36
151	$D_3^3$	$P3_1^{12}$	37
152	$D_3^4$	$P3_1^{21}$	37
153	$D_3^5$	$P3_2^{12}$	37
154	$D_3^6$	$P3_2^{21}$	37
155	$D_3^7$	R32	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
156	$C_{3v}^1$	P3m1	33, 36
157	$C_{3v}^2$	P31m	33, 35
158	$C_{3v}^3$	P3c1	34, 35
159	$C_{3v}^4$	P31c	34, 36

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160	$C_{3v}^5$	R3m	1, 4, 5, 7, 9
161	$C_{3v}^6$	R3c	2, 3, 6, 8, 10
162	$D_{3d}^1$	$\bar{P}31m$	33, 35
163	$D_{3d}^2$	$\bar{P}31c$	34, 36
164	$D_{3d}^3$	$\bar{P}3m1$	33, 36
165	$D_{3d}^4$	$\bar{P}3c1$	34, 35
166	$D_{3d}^5$	$\bar{R}3m$	1, 4, 5, 7, 9
167	$D_{3d}^6$	$\bar{R}3c$	2, 3, 6, 8, 10
168	$C_6^1$	P6	33, 34
169	$C_6^2$	$P6_1$	37
170	$C_6^3$	$P6_5$	37
171	$C_6^4$	$P6_2$	37
172	$C_6^5$	$P6_4$	37
173	$C_6^6$	$P6_3$	35, 36
174	$C_{3h}^1$	$\bar{P}6$	33, 34, 35, 36
175	$C_{6h}^1$	$P6/m$	33, 34

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176	$C_{6h}^2$	$P6_3/m$	35, 36
177	$D_6^1$	$P622$	33, 34
178	$D_6^2$	$P6_122$	37
179	$D_6^3$	$P6_522$	37
180	$D_6^4$	$P6_222$	37
181	$D_6^5$	$P6_422$	37
182	$D_6^6$	$P6_322$	35, 36
183	$C_{6v}^1$	$P6mm$	33
184	$C_{6v}^2$	$P6cc$	34
185	$C_{6v}^3$	$P6_3cm$	35
186	$C_{6v}^4$	$P6_3mc$	36
187	$D_{3h}^1$	$\bar{P}6m2$	33, 36
188	$D_{3h}^2$	$\bar{P}6c2$	34, 35
189	$D_{3h}^3$	$\bar{P}62m$	33, 35
190	$D_{3h}^4$	$\bar{P}62c$	34, 36
191	$D_{6h}^1$	$P6/mmm$	33



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192	$D_{6h}^2$	P6/mcc	34
193	$D_{6h}^3$	$P6_3/mcm$	35
194	$D_{6h}^4$	$P6_3/mmc$	36
195	$T^1$	P23	1, 2, 3, 4
196	$T^2$	F23	5, 6, 7, 8
197	$T^3$	I23	9
198	$T^4$	$P2_13$	11
199	$T^5$	$I2_13$	10
200	$T_h^1$	Pm3	1, 3
201	$T_h^2$	Pn3	2, 4
202	$T_h^3$	Fm3	5, 6
203	$T_h^4$	Fd3	7, 8
204	$T_h^5$	Im3	9
205	$T_h^6$	Pa3	11
206	$T_h^7$	Ia3	10
207	$O^1$	P432	1, 2

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208	$O^2$	$P4_2 32$	3, 4
209	$O^3$	F432	5, 6
210	$O^4$	$F4_1 32$	7, 8
211	$O^5$	I432	9
212	$O^6$	$P4_3 32$	11
213	$O^7$	$P4_1 32$	11
214	$O^8$	$I4_1 32$	10
215	$T_d^1$	$\bar{P}43m$	1, 4
216	$T_d^2$	$\bar{F}43m$	5, 7
217	$T_d^3$	$\bar{I}43m$	9
218	$T_d^4$	$\bar{P}43n$	2, 3
219	$T_d^5$	$\bar{F}43c$	6, 8
220	$T_d^6$	$\bar{I}43d$	10
221	$O_h^1$	Pm3m	1
222	$O_h^2$	Pn3n	2
223	$O_h^3$	Pm3n	3

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225	$O_h^5$	Fm3m	5
226	$O_h^6$	Fm3c	6
227	$O_h^7$	Fd3m	7
228	$O_h^8$	Fd3c	8
229	$O_h^9$	Im3m	9
230	$O_h^{10}$	Ia3d	10

















































































































