UPPER BOUNDS ON THE MAGNETO-ELECTRIC SUSCEPTIBILITY

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The positive-definiteness of the electric susceptibility tensor at fixed magnetization provides an upper bound on the magneto-electric susceptibility.

Recently Brown et al. [1] have found an upper bound on the magneto-electric susceptibility. Their method of proving this and their result both suggest that what they find is a property of the electric susceptibility. When this is realized it is also possible to take the symmetry more fully into account than is done in ref. [1].

As starting point we may take the relations

$$P_i = \epsilon_0\kappa^H_{ik}E_k + \alpha_0\alpha_{ik}H_k \qquad (1)$$

$$M_i = \alpha_0\alpha_{ki}E_k + \mu_0\chi^E_{ik}H_k\,. \qquad (2)$$

Here κ^H_{ik} is the electric susceptibility at fixed magnetic field, χ^E_{ki} the magnetic susceptibility at fixed electric field, and α_{ik} the magneto-electric susceptibility at fixed magnetic - or electric-field. (It is well known that $\alpha_{ik} = \alpha_{ki}$.) The constant α_0 is $1/c$ so that all susceptibilities are dimensionless.

Eqs. (1) and (2) may be transformed in various ways. Here we are interested only in the electric susceptibility at constant magnetization, κ^M_{ik}. This quantity appears in the equation

$$P_i = \epsilon_0(\kappa^H_{ik} - \alpha_{ij}\alpha_{ks}\psi_{js})E_k + (\epsilon_0/\mu_0)^{\frac{1}{2}}\alpha_{ij}\psi_{jk}M_k \qquad (3)$$

where ψ_{jk} denoted the inverse of the magnetic susceptibility tensor:

$$\psi_{ij}\chi^E_{jk} = \delta_{ik} = \chi^E_{ij}\psi_{jk}\,. \qquad (4)$$

Thus

$$\kappa^M_{ik} = \kappa^H_{ik} - \alpha_{ij}\alpha_{ks}\psi_{js}\,. \qquad (5)$$

It is positive-definiteness of this tensor that is in fact proved in ref. [1].

If furthermore one supposes, as was done in ref. [1], that the tensor χ^E_{ik} is positive -definite, one deduces from eq. (5) inequalities that in most cases have the form given in ref. [1], viz.:

$$\alpha^2_{ik} < \kappa_{ii}\chi_{kk}\,.$$

However, in the case of the symmetries 222, $2m'm'$, $m'm'm$; $2'm'$, $2/m'$; $2'$, m, $2'/m$ (and of course 1 and $\bar{1}'$), more complicated inequalities occur. Thus, for 222, $2m'm'$ and $m'm'm'$, one finds:

$$\alpha_{11} < \frac{\kappa_{11}}{\chi_{33}}(\chi_{11}\chi_{33} - \chi_{13}\chi_{31})$$

$$\alpha^2_{22} < \kappa_{22}\chi_{22}$$

$$\alpha^2_{33} < \frac{\kappa_{33}}{\chi_{11}}(\chi_{11}\chi_{33} - \chi_{13}\chi_{31}\,.$$

A more detailed discussion will be published elsewhere.

References

1. W. F. Brown Jr., R. M. Hornreich and S. Shtrikman, Phys. Rev. 168 (1968) 574.

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