AN EXPLANATION OF ANOMALOUS OPTICAL BEHAVIOUR OF THE IMPROPER FERROELECTRIC $\operatorname{Gd}_2(\operatorname{MoO}_4)_3$

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Typical features of the electric field dependences of an order parameter and of polarization in improper ferroelectrics are discussed. The anomalous optical hysteresis observed in the improper ferroelectric $\mathrm{Gd}_2(\mathrm{MoO}_4)_3$ is shown to be explained by them.

A ferroelectric phase does not necessarily result from a phase transition having the polarization as order parameter (Indenbom[1]). Boracites [2] are an example of such a ferroelectric, KDP and GMO are others. Dvořák [3-5] has called such ferroelectric improper and has given a phenomenological theory for them. Although correct in principle, it still does not reveal some of the peculiar features of these crystals. One of the authors (J. K.) [6] has therefore proposed a modification that permits a classification of the improper ferroelectrics according to the strength of the coupling between the polarization and the order parameter and explains the anomalous dielectric and electro-mechanical properties of some improper ferroelectrics. Here we wish to show that the unusual optical behaviour of GMO, found recently by Nakamura and Kumada [7], can be explained bt the same theory.

For our present purposes the following approximation to the free energy of the crystal will be sufficient

$$\begin{split} A &= \frac{1}{2} B (T - T_{\theta}) \theta_{3}^{2} + \frac{1}{4} \gamma \theta_{3}^{4} + \frac{1}{2} \omega P_{3}^{2} \\ &+ \frac{1}{2} s_{11} (X_{1}^{2} + X_{2}^{2}) - \frac{1}{2} b_{31} (X_{1} - X_{2}) P_{3} \\ &- Q_{31} (X_{1} + X_{2}) P_{3}^{2} - \frac{1}{2} h_{31} (X_{1} - X_{2}) \theta_{3} \end{split}$$

$$-R_{31}(X_1 + X_2)\theta_3^2 + f\theta_3 P_3.$$

The temperature-dependent order parameter θ_3 is coupled to the stresses X_1 and X_2 and, by a negative constant f, to the polarization; ω , the reciprocal dielectric permittivity, and γ are positive, and the transition is of second order. The equilibrium conditions for the machanically free crystal are

$$E_3 = f\theta_3 + \omega P_3, \qquad \theta = \beta \left(T - T_\theta\right) + \gamma \theta_3^3 + f P_3 \ .$$

These equations have non-zero spontaneous solutions $\theta_{\rm S}$, $P_{\rm S}$ if $T < T_{\rm O} = T_{\theta} + (f^2/\omega\beta)$.

The characteristic features of an improper ferroelectric depend directly on the magnitude of f/ω . For small values of f/ω , peculiar properties should be observed; boracites and GMO are in this case. Of course, $P_{\rm S}\ll\theta_{\rm S}$, and also the θ - and P- permittivities in weak electric fields

$$\eta_{\theta} = -\frac{f}{\omega} \frac{1}{\beta (T - T_{0})}, \qquad \eta_{\mathrm{P}} = \frac{1}{\omega} + \frac{f^{2}}{\omega^{2}} \frac{1}{\beta (T - T_{0})}$$

have sharp peaks in the close vicinity of T_0 . The *E*-dependence of θ_3 and P_3 are given in fig. 1.

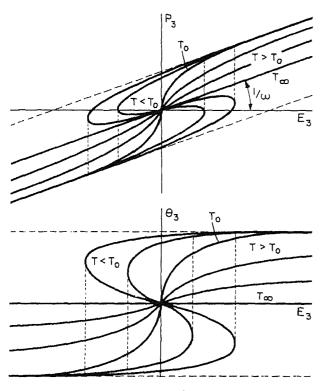


Fig. 1. Hysteresis of θ_3 and of P_3 .

The θ_3 - E_3 hysteresis loop is square, with field independent saturation; the P_3 - E_3 loop is not square and the saturation line has a positive slope: $1/\omega$. Above $T_{\rm O}$, the field dependence of θ_3 disappears immediately, while P_3 = $(1/\omega)E_3$. This different field dependence must manifest itself in an unusual behaviour of all physical properties that can be expressed as a difference of terms depending on θ_3 and terms depending on P_3 . A typical example is the temperature dependence of pure shear in Fe-I-boracite [8]. The birefringence of GMO is another example.

According to Nakamura et al., the spontaneous birefringence Δn_{xy}^{S} perpendicular to the polar axis decreases with an increase of E_3 (the direction of $P_{\rm S}$ being the + direction). Now

$$\begin{split} \Delta n_{xy} &= n_0^3 \{ (r_{13}' + b_{31} \Pi_{44}) P_3 \\ &+ (u_{13}' + h_{31} \Pi_{44}) \theta_3 \} = \Delta n_P + \Delta n_{\ell\ell} \end{split} .$$

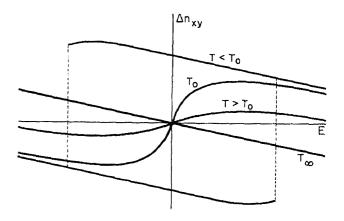


Fig. 2. Hysteresis of $\Delta n_{\chi \chi}$.

Here n_0^3 is the ordinary refractive index, r_{13} is an electro-optic coefficient of the clamped crystal, u_{13} the corresponding coefficient for θ_3 , and $1\bar{l_{44}}$ an electro-elastic coefficient. These coefficients are approximately independent of temperature; $\Delta n_{\chi y}$ is the sum of Δn_{θ} and $\Delta n_{\mathbf{p}}$, terms proportional to θ_3 and P_3 respectively. If Δn_{θ} and $\Delta n_{
m P}$ have opposite signs and $|\Delta n_{
m A}|$ $> |\Delta n_{\mathbf{p}}|$, which is quite a plausible condition, then Δn_{xy} depends on E_3 as shown in fig. 2, where an arbitrary small value of f has been chos..n. Such an unusual optical hysteresis is observed in GMO and must always occur in the conditions assumed here; more extensive measurements of $\Delta n_{\chi y}$ both in para-electric and ferroelectric states would reveal the features shown in fig. 2.

References

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